HOMI BHABHA CENTRE FOR SCIENCE EDUCATION
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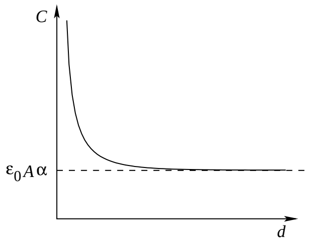
Indian National Physics Olympiad - 2013
Solutions

PLEASE NOTE THAT ALTERNATE/EQUIVALENT SOLUTIONS MAY EXIST. Brief solutions are given below.

1. (a) $C = \dfrac{\epsilon_0 A\alpha}{1 - e^{-\alpha d}}$

   (b)



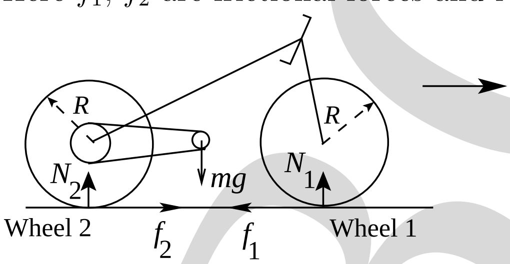
   (c) Charge $= \dfrac{\epsilon_0 A\alpha V}{1 - e^{-\alpha d}}$

   (d) $\vec{E}(x) = \dfrac{\alpha V e^{-\alpha x}}{1 - e^{-\alpha d}}\,\hat{x}$

2. (a) $\phi = \sin^{-1}\left(\dfrac{nh}{d\sqrt{2m_0K}}\right)$

   (b) $d \approx 2.4$ Å

3. (a) Here f_1, f_2 are frictional forces and N_1, N_2 are normal reactions.



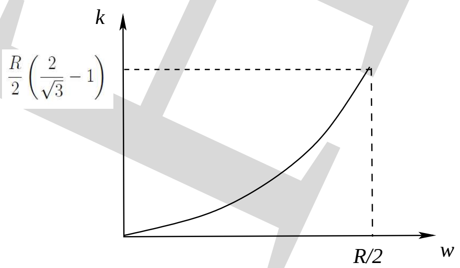
   (b) $a = \dfrac{\tau}{MR^2 + 2I}R$

   (c) $a \leq \dfrac{\mu g/2}{\left(1 - \dfrac{\mu}{4}\right)}$

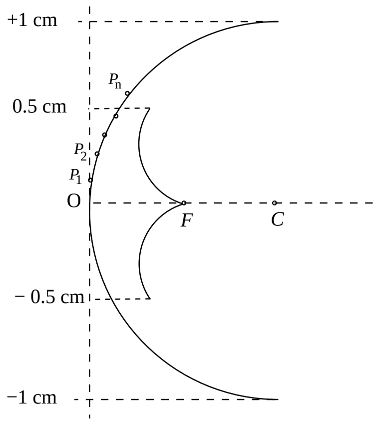
   (d) $a_m = 2g/3$

4. (a) $k = \dfrac{R}{2}\left[\dfrac{R}{(R^2 - \omega^2)^{1/2}} - 1\right]$

   (b)



(c)



5. (a) Torque $= \dfrac{2\mu_0 I^2 (a^2+b^2) abd \sin\phi}{\pi\left[(a^2-b^2)^2 + 4a^2b^2\sin^2\phi\right]}\,\hat{z}$

(b) $\phi = \alpha;\ -\alpha;\ \pi-\alpha;\ \pi+\alpha$
where $\alpha = \sin^{-1}\left[(a^2-b^2)/2ab\right]$

6. (a) $g(r,\theta,\dot{r},\dot{\theta}) = \dfrac{F(r)}{m} + r\dot{\theta}^2$

(b) From null azimuthal component

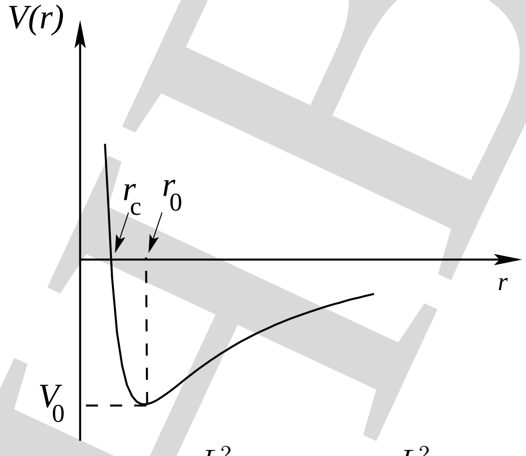
$$\begin{aligned} 2\dot{r}\dot{\theta} + r\ddot{\theta} &= 0 \\ \frac{d}{dt}(r^2\dot{\theta}) &= 0 \\ \frac{d}{dt}(mr^2\dot{\theta}) &= 0 \\ \frac{d}{dt}L &= 0 \end{aligned}$$

$\Rightarrow L =$ constant

(c) $E = \dfrac{m(\dot{r}^2 + r^2\dot{\theta}^2)}{2} - \dfrac{GMm}{r}$

(d) $V(r) = \dfrac{L^2}{2mr^2} - \dfrac{GMm}{r}$

(e)



where $r_0 = \dfrac{L^2}{GMm^2}$, $r_c = \dfrac{L^2}{2GMm^2}$ and $V_0 = -\dfrac{G^2M^2m^3}{2L^2}$

(f) For $E > 0$ orbit is hyperbola. For $E < 0$ orbit is ellipse. At $r = r_o$ orbit is circular.

(g) $r_0 = \dfrac{L^2}{GMm^2}$

(h) $T_r = 2\pi\sqrt{\dfrac{r_0^3}{GM}}$

(i) $n > -3$

7. (a) $\Gamma_a = \dfrac{(1-\gamma_a)m_a g}{\gamma_a R}$ or $-\dfrac{m_a g}{C_a}$
where $\gamma_a = 7/5$ (ratio of specific heats at constant pressure and volume) and C_a is molar specific heat at constant pressure for air.
$|\Gamma_a| \approx 0.01°\mathrm{K}\cdot\mathrm{m}^{-1}$

(b) $\Gamma_b = -\dfrac{m_a g T_b}{C_b T_a}$

(c) $\ddot{z} = g\left(\dfrac{m_a T_b}{T_a m_b} - 1\right)$

(d) $z_0 = \dfrac{T_0}{\Gamma_a}\left[1 - \left(\dfrac{m_b}{m_a}\right)^{1/(\eta-1)}\right]$
where $\eta = C_a/C_b$.

(e) Condition: $C_a > C_b$
$\omega = g\sqrt{\dfrac{m_a(\eta-1)}{C_a T_0}\left(\dfrac{m_a}{m_b}\right)^{1/(\eta-1)}}$

(f) $\tau \approx 95$ s

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