Astronomy

1) **(a)** Draw a figure indicating the relative positions yourself to verify the answer.

2) **(b)** We obtain $x_1 + x_2 = A\sin\left(\omega t + \frac{\pi}{3}\right)$. Now, we require $x_1 + x_2 + x_3 = 0$, so that $x_3 = -(x_1 + x_2) = -A\sin\left(\omega t + \frac{\pi}{3}\right) = A\sin\left(\omega t + \frac{\pi}{3} + \pi\right)$. Comparing this with the given expression for x_3, we get the result.

3) **(d)** Express $\tan^2\theta$ in terms of $\sec^2\theta$ and obtain a quadratic equation in $\sec\theta$. This gives two values of θ satisfying the condition given in option (d).

4) **(d)**

5) **(b)** We have $\tan\alpha = \frac{q\sin\theta}{p + q\cos\theta}$ and $\tan\beta = \frac{q\sin\theta}{p - q\cos\theta}$. Note that the angle between $\vec{p}$ and $(-\vec{q})$ is $(180 - \theta)$. Add the expressions to get the result.

6) **(d)** Use the expressions for different quantities to get the result. The physical quantity is called skin depth – distance to which electromagnetic wave penetrates into a body.

7) **(b)** Red giant stars have low surface temperatures and high luminosity.

8) **(b)**

9) **(b)** Express $\tan 65^0 = \tan(45^0 + 20^0)$ and $\tan 25^0 = \tan(45^0 - 20^0)$. Use formula for $\tan(A + B)$. Also note that angles 20^0 and 70^0 as well as 55^0 and 35^0 form pairs of complementary angles.

10) **(c)** Note that units of μ_0 are henry/m whereas those of ε_0 are farad/m. Then we can write $\sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{L\omega}{C\omega}} = \sqrt{\left(\frac{1}{\omega C}\right)(\omega L)}$. Note that unit of each of the quantities in bracket is ohm and hence the answer.

11) **(c)** The spectral type depends on the surface temperature of the star. The Sun has temperature of 6000 K, which is typical for G class.

12) **(a)** Refer to the figure and write the initial potential energy of the system as $U_1 = \frac{10^2}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{30} + \frac{q_1 q_3}{40} + \frac{q_2 q_3}{50}\right]$. Similarly the expression for the final potential

energy of the system is $U_2 = \frac{10^2}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{30}+\frac{q_1q_3}{40}+\frac{q_2q_3}{10}\right]$. Get the difference between the two potential energies and compare it with the given expression to get the answer.

13) (c) The maximum number of eclipses (both solar and lunar) that can occur in a year is 7. However, the combination of solar and lunar eclipses can be either 4 and 3 or 5 and 2 respectively.

14) (c) We get $A \cap B = \{6,12,18......396\}$. The last term, that is, n th term in this set can be written as 6 + (n –1) = 396, giving n = 66. This gives $n(A \cap B) = 66$. Again use $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ to get the result.

15) (a)

16) (b) Let the speeds of points P and Q be $\frac{dy}{dt}$ and $\frac{dx}{dt}$ respectively, where $y = l \sin\theta$ and $x = l \cos\theta$. Obviously, $\frac{dy}{dt} = -l\cos\theta\frac{d\theta}{dt}$ (negative sign indicating the decrease of y with increase of time t) and $\frac{dx}{dt} = -l\sin\theta\frac{d\theta}{dt}$. This gives $\frac{dx/dt}{dy/dt} = \tan\theta$. With $\frac{dy}{dt} = \sqrt{3}$ and $\tan 30^\circ = \frac{1}{\sqrt{3}}$, we get the answer.

17) (a) Note that the amplitude of ray **a** will be r. Therefore, the intensity of the ray entering into the film is $(1 - r^2)$ and its amplitude is $(1 - r^2)^{1/2}$. Then , the amplitude of the ray getting reflected internally from the lower surface of the film is $r \times (1 - r^2)^{1/2}$. Thus, the intensity of the ray a' coming out if the film from the lower surface is $[(1 - r^2)^{1/2}]^2 - [r \times (1 - r^2)^{1/2}]^2 = (1 - r^2) - (1 - r^2)r^2 = (1 - r^2)^2$. Therefore, its amplitude is $(1 - r^2)$.

18) (c) Refer to any standard text book on Optics.

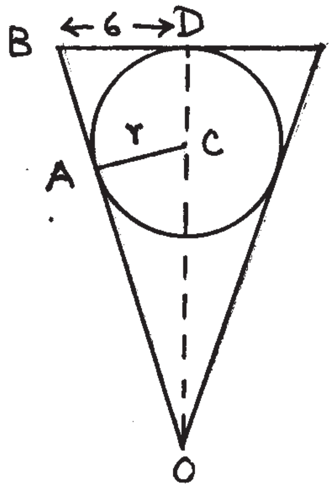
19) (c) Let $T_5 = a$, $T_3 = a - 2d$ and $T_7 = a + 2d$, so that $T_1 = a - 4d$. Use the data to get $a = 3$ and $d = \pm \frac{1}{2}$. This gives two possible values of T_1 and then $S_{16} = 76$ or 20.

20) (a) Time (t_1) required to cover the height of 0.40 m upto the top of the window and that (t_2) required to cover a total of (0.40 + 0.50) m upto the bottom of the window can be determined. Taking the difference ($t_2 - t_1$) gives the answer.

21) (a) The circle has centre $C\left(\frac{3}{4},0\right)$ and radius $r=\frac{3}{4}$. The distance of point P(–2, 1) from C is $\frac{\sqrt{137}}{4}$. The nearest point, say A, from P that lies on the circle must be along line PC. Distance PA is obviously greater than 2 and hence no point on the circle can have distance 2 from P.

22) (b) Before opposition the earth is about to overtake the outer planet and hence the planet appears to move in opposite position.

23) (a) Refer to the figure.



Triangles BDO and CAO can be easily proved to be similar, so that AC : OC = BD : OB. Therefore, $\frac{r}{8-r}=\frac{6}{10}\Rightarrow r=3$. Hence the required fraction of volume is $\dfrac{\frac{4}{3}\pi(3)^3}{\frac{1}{3}\pi(6)^2(8)}=\frac{3}{8}$.

24) (b)

25) (c)

26) (d) We can write

$$\left(\frac{x-3}{x+2}\right)^x=\left(1-\frac{5}{x+2}\right)^x=\left(1-\frac{5}{x+2}\right)^{(x+2)-2}=\left(1-\frac{5}{x+2}\right)^{x+2}\times\frac{1}{\left(1-\frac{5}{x+2}\right)^2}.$$

Taking the limit as $x\to\infty$, we get the result.

27) **(a)** The latitude of Amritsar is greater than 23.5° North and hence the Sun never comes at the zenith.

28) **(a)** Expand $(a - b)^2$ and similarly $(b - c)^2$, $(c - d)^2$ and $(d - a)^2$. Add all these and note that the sum of these squares is ≥ 0. Simplify the inequality to get the result.

29) **(a)** Distance between the foci = 2 ae = 2, so that ae = 1. Also the sum of the focal distances of P is $2a$ giving $a = \sqrt{2}+1$. This gives $e = \frac{ae}{a} = \frac{1}{\sqrt{2}+1} = \sqrt{2}-1$.

30) **(c)** As per International Astronomical Union's conventions, constellation Ophiuchus occupies portion between Scorpius and Sagittarius.

31) **(c)** Let α and α^2 be the roots of the quadratic equation. Using the expressions for the sum and the product of the roots of the quadratic equation, we get $\alpha = 1$. This gives the final result.

32) **(b)** Consider the dot product of the two vectors $(\vec{a}+2\vec{b})$ and $(5\vec{a}-4\vec{b})$ which is zero. Simplify this to get $\cos\theta = 0.5$ and hence the result.

33) **(c)**

34) **(c)** At Mercury's inferior conjunction if the earth is at Mercury's nodal plane, transit occurs. The earth comes at Mercury's nodal plane in the months of May and November.

35) **(c)** Pisces will be at 90° west of Gemini.

36) **(d)** We have $3A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, $A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix}$. Also note that $A + A + A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = A^2$.

37) **(d)** Red shift can be calculated as (observed wavelength – rest wavelength) divided by rest wavelength.

38) **(b)** Consider the equation $x^3 - 64 = 0$, factorize it. Solve the quadratic factor to get the solution.

39) **(a)** Use the usual expression for the wavenumber of a spectral line in Balmer series. For hydrogen atom with $Z = 1$, $\frac{1}{\lambda_H} = \frac{5R}{36}$. Again for singly ionized helium

atom with $Z = 2$, $\frac{1}{\lambda_{He^+}} = \frac{3R}{4}$. Knowing the wavelength of the first line in Balmer series for hydrogen atom, we get the answer.

40) (b) We have $x^{x\sqrt{x}} = \left(x^{\frac{3}{2}}\right)^x \Rightarrow x^{x\sqrt{x}} = x^{\frac{3x}{2}} \Rightarrow x\sqrt{x} = \frac{3x}{2} \Rightarrow \sqrt{x} = \frac{3}{2} \Rightarrow x = \frac{9}{4} = 2.25$

41) (b) Note that the electrostatic potential energy is equal to the energy of the alpha particle. Thus, $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = 10\,\text{MeV}$, where r can be taken as the distance closest approach.

42) (b) Electrostatic potential will add being a scalar but the electric field will get added to zero being a vector. Magnetic field will obviously not be zero.

43) (d) Note that the current in the circuit is 2 A which can be determined from the voltage across the resistance. This indicates that the voltage across the capacitor is 90 volt. The voltages across capacitor and inductor are out of phase by 180^0 and their resultant ha to be 90 volt, so as to satisfy $150^2 = 120^2 + (V_L - V_C)^2$. This give the voltage across as 180 volt.

44) (b) The given frequencies happen to be the odd multiples of the fundamental frequency 85 Hz. The corresponding wavelength is $4l$ where l is the resonating length of the pipe open at one end and closed at the other.

45) (d) Write $m = 6 + \sqrt{35} = 6 + 2\sqrt{\frac{35}{4}} = \frac{7}{2} + \frac{5}{2} + 2\sqrt{\frac{7}{2} \times \frac{5}{2}}$, so that $\sqrt{m} = \sqrt{\frac{7}{2}} + \sqrt{\frac{5}{2}}$.

Similarly, $\sqrt{n} = \sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}$. This gives the required result.

46) (d) At the hole, the pressure outside the tube is ρgh. While from inside the tube pressure is $\rho g(h - h')$ which is less than that outside and hence mercury will not come out at all.

47) (d) We have $\alpha^3 = \beta^3 = 1$. Also $\alpha + \beta = -1$ and $\alpha\beta = 1$. We can also write $\alpha^2 + \alpha = \beta^2 + \beta = -1$. Take the product AB and use these results.

48) (d) Draw the figure yourself and check that moment of inertia of side PS is $\frac{m}{6}\left(\frac{l}{3}\right)^2$. Also the moment of inertia of each of the sides PQ and RS is

$\frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2$. Adding all these one gets the answer.

49) (c) Note that the focal length of the combination of lenses is given by the relation $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$. Also use lens makers formula for focal length of each of the lenses and get the answer.

50) (c) Use superposition theorem. When dc source of 6 V is shorted, the current through 10 ohm resistance is $\frac{2}{7}$ A from B to A. Whereas when 10 V dc source is shorted, the current happens to be $\frac{3}{35}$ A from A to B. Adding the two currents with appropriate sense, we get the result.

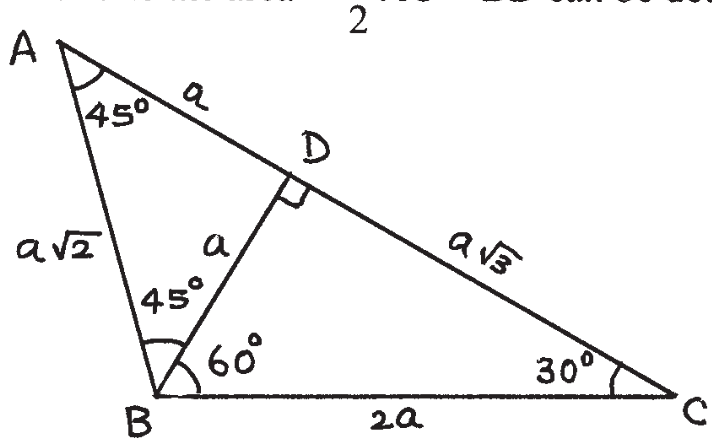
51) (a) Write $(1+x^2)^3 = 1+3x^2(1+x^2)+x^6 = 8x^3\cos^3\theta$. Simplifying this we get $\frac{x^6+1}{x^3}+\frac{3(2x\cos\theta)}{x} = 8\cos^3\theta$. Using the expression for cos (3θ), we get the result.

52) (b) Let the upthrust acting on the metal sphere at 0°C be $F_0 = V_0\rho_{w0}g$ and that at 50°C be $F_{50} = V_{50}\rho_{w50}g$. Then, $\frac{F_{50}}{F_0}=\frac{V_{50}\rho_{w50}g}{V_0\rho_{w0}g}=\frac{(1+\gamma_m\Delta\theta)}{(1+\gamma_w\Delta\theta)}$. Since $\gamma_m < \gamma_w$, we have $F_{50} < F_0$, that is upthrust at 50°C is smaller than that at 0°C. Hence the apparent weight $W_1 < W_2$.

53) (b) Beat frequency $= f_1 - f_2 = \frac{v}{2l}-\frac{v}{2(l+x)} = \frac{v}{2l}\left[1-\left(1+\frac{x}{l}\right)^{-1}\right] = \frac{v}{2l}\frac{x}{l}$ (neglecting higher order terms in $\frac{x}{l}$).

54) (c) Current in the circuit at an instant $i = \frac{10 - L\frac{di}{dt}}{20}$. Since the current is decreasing with increasing resistance, $\frac{di}{dt}$ is negative and the term in the numerator adds to 10 volt. Therefore, the current is greater than 0.5 A.

55) (a) Refer to the figure. The largest side = $a + \sqrt{3}a = a(1+\sqrt{3}) = 1+\sqrt{3}$ (given), implies $a = 1$. With this the area = $\frac{1}{2}$AC × BD can be determined.



56) (b) We have with usual notation $pV = nRT_1$ and $p\left(\frac{5V}{2}\right) = nRT_2$. Again, change in internal energy $\Delta U = nC_v dT = n\frac{R}{\gamma - 1}(T_2 - T_1) = \frac{nRT_2 - nRT_1}{\gamma - 1}$. Using the above relations we get the result.

57) (b) Use the usual relations, energy $E \propto \frac{1}{n^2}$, linear momentum $p \propto \frac{1}{n}$ and angular frequency $\omega \propto \frac{1}{n^3}$ where n is the principal quantum number. Use this to get the answer.

58) (b) Let $\sin^{-1} x = \alpha$ and $\sin^{-1} y = \beta$, so that we have $\sin\alpha = x, \cos\alpha = \sqrt{1-x^2}$ and $\sin\beta = y, \cos\beta = \sqrt{1-y^2}$. From this we write expression for $\sin(\alpha+\beta)$ and get the final result.

59) (d) Take self dot products for $(\vec{A}+\vec{B}) = \vec{C}$ and $(\vec{A}-\vec{B}) = \vec{D}$. Add these to get the result.

60) (b) The given expression $= (1+\cot\theta)^2 - \sec^2\left(\theta + \frac{\pi}{2}\right)$. Also $\tan\left(\theta + \frac{\pi}{2}\right) = -\cot\theta$. With this on simplification we get the result.

61) (c) Since the charge is stationary, its speed is zero and hence no force is exerted by the magnetic field.

62) (a) The centres and radii of the two circles respectively are (0,0), (3,4) and 2,7. The distance between the centres is 5 which is also the difference between the radii. This means that the two circles touch each other internally and hence only one common tangent.

63) (c) Note that force constant is inversely proportional to length, so that the force constants of the two pieces are $k_1 = \frac{5}{2}k$ and $k_2 = \frac{5}{3}k$. The effective force constant in this case is $k' = k_1 + k_2 = \frac{25k}{6}$. This give the periodic time T' using the usual relation, $T' = 2\pi\sqrt{\frac{m}{k'}}$.

64) (a) The Cartesian coordinates corresponding to polar coordinates (r, θ) of a point are given by $x = r\cos\theta$, $y = r\sin\theta$ so that $r^2 = x^2 + y^2$. From the given equation we can get $r = x - 8 \Rightarrow r^2 = (x-8)^2 \Rightarrow x^2 + y^2 = x^2 - 16x + 64$ which yields an expression $y^2 = -16(x-4)$ representing a parabola.

65) (b) Let the linear mass density be $\lambda = k(l - x)$, so that the mass element $dm = \lambda\, dx$ $= k(l - x)\, dx$. The X coordinate of centre of mass can be determined by using the usual formula. The Y coordinate is obviously zero.

Q. 66. None of the given alternatives is correct.

67) (d) Use an expression for the area of a triangle in terms of a determinant. Therefore,

$$\text{Area of } \Delta\text{ ABC} = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} a & a+b+c & 1 \\ b & a+b+c & 1 \\ c & a+b+c & 1 \end{vmatrix} = \frac{1}{2}(a+b+c)\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

68) (b) Let $n(X) = 100a$ be the population of the village. Let A be the set of persons knowing Hindi and B be the set of people knowing English. Then, the number of people knowing Hindi or English or both $= n(A \cup B)$. Thus, the number of people who can understand only Marathi is $n(X) - n(A \cup B)$, which is given to be $52a$.

Therefore, $100a - n(A \cup B) = 52a \Rightarrow n(A \cup B) = 48a \Rightarrow n(A) + n(B) - n(A \cap B)$ $= 48a \Rightarrow 35a + 23a - n(A \cap B) = 48a. \Rightarrow n(A \cap B) = 10a = 1500$ (given). This gives $a = 150$, so that population of the village is $n(X) = 100a = 15000$.

69) (c) We can write $S = \frac{3}{2} + \frac{1}{2} + \frac{1}{4} + \frac{3}{20} + \frac{1}{10} ..... + \frac{1}{850}$

$$= 3\left[\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} ..... + \frac{1}{2550}\right] = 3\left[\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} .... + \frac{1}{50\times 51}\right]$$

which can be further written as $3\sum_{1}^{50}\frac{1}{r(r+1)} = 3\sum_{1}^{50}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 3\left(\frac{1}{1} - \frac{1}{51}\right)$. This can then be simplified to get the answer

70) (c) We have $\frac{1}{12!} - \frac{1}{14!} = \frac{13\times 14}{14!} - \frac{1}{14!} = \frac{182-1}{14!} = \frac{181\times 15}{15\times 14!} = \frac{2715}{15!} = \frac{x}{15!}$ (given) and therefore the result.

71) (a) For a tangent to be parallel to the X axis, the slope of the curve at a point must be zero. Differentiate the equation and put $\frac{dy}{dx} = 0$. Factorize the expression to get only one real value of $x = -1$ and hence the result.

72) (a) Let $5^{3x-4} = a$ so that $5^{4-3x} = \frac{1}{a}$. Adding the expressions and using the data we get $a + \frac{1}{a} = 2 \Rightarrow (a-1)^2 = 0 \Rightarrow a = 1$. With this we get $5^{3x-4} = 1 \Rightarrow 3x - 4 = 0$ and hence the result.

73) (d) Given that $f(x) = \sin x \cos x = \frac{1}{2}\sin 2x \Rightarrow f'(x) = \cos 2x$. Note that $f(x)$ is increasing when $f'(x) > 0 \Rightarrow \cos 2x > 0 \Rightarrow x \in \left(0, \frac{\pi}{4}\right)$ or $x \in \left(\frac{3\pi}{4}, \pi\right)$. Hence the correct option is (d).

74) (c) We can write $b^n = 16807$ ....(i), $n\, a\, b^{n-1} = 48020$ ...(ii) and $\frac{n(n-1)}{2} a^2 b^{n-2} =$ 54880 ...(iii). Equation (i) ÷ equation (ii) gives $\frac{b}{na} = \frac{16807}{48020} = 0.35$ ....(iv)

Similarly, equation (ii) ÷ equation (iii) gives $\frac{2b}{(n-1)a} = \frac{48020}{54880} = 0.875$ ....(v).

Equation (iv) ÷ equation (v) gives $\frac{n-1}{2n} = 0.4 \Rightarrow n = 5$. Using eq(i), we get $b = 7$ and eq(iv) gives $a = 4$.

75) (a) With usual notations, the excess pressures for the two bubbles are $p_A = \frac{4T}{R_A}$ and $p_B = \frac{4T}{R_B}$ respectively, obviously giving $p_A > p_B$. Now, the excess pressure across the interface is $p_A - p_B = \frac{4T}{R}$ where R is the radius of curvature of the interface. Substituting the values we get the answer.

76) (d) Use the expression for the total energy $= \frac{1}{2}ka^2$ to get the value of k and use it to find the periodic time.

77) (a) Let I = $\int_0^{\frac{\pi}{6}} \frac{\cos^4(3x)}{\sin^4(3x)+\cos^4(3x)}dx$ .... (i).

Also

$$I = \int_0^{\frac{\pi}{6}} \frac{\cos^4\left[3\left(\frac{\pi}{6}-x\right)\right]}{\sin^4\left[3\left(\frac{\pi}{6}-x\right)\right]+\cos^4\left[3\left(\frac{\pi}{6}-x\right)\right]}dx = \int_0^{\frac{\pi}{6}} \frac{\cos^4\left(\frac{\pi}{2}-3x\right)}{\sin^4\left(\frac{\pi}{2}-3x\right)+\cos^4\left(\frac{\pi}{2}-3x\right)}dx$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin^4(3x)}{\cos^4(3x)+\sin^4(3x)}dx \text{ ..... (ii)}$$

Adding equations (i) and (ii), we get

$$2\,I = \int_0^{\frac{\pi}{6}} (1)dx = \frac{\pi}{6} \Rightarrow \text{given integral } I = \frac{\pi}{12}$$

78) (b) Refer to the truth tables of NOT, OR and AND gates and verify that the outputs come out to be 1 and 0 respectively.

79) (d) From Bohr theory of hydrogen atom we have $mvr = n\frac{h}{2\pi}$, so that linear momentum of the electron is $mv = \frac{nh}{2\pi r}$. Use this in the expression for de Broglie wavelength $\lambda = \frac{h}{mv}$ to get the answer.

80) (d) Given integral can be rewritten as

$$I = \frac{1}{5}\int \frac{5x^4 + 5(\log 5)(5^{x-1})}{5^x + x^5}dx = \frac{1}{5}\int \frac{5x^4 + (\log 5)(5^x)}{5^x + x^5}dx = \frac{1}{5}\int \frac{\frac{d}{dx}(5^x + x^5)}{5^x + x^5}dx$$

$$= \frac{1}{5}\log(5^x + x^5)$$