SCRA 2015
MATHEMATICS

1. If $\int \frac{g''(x)\, g(x)dx}{\{g'(x)\}^2} = x + \text{constant}$, then the function g(x) will be of the form
   (a) $ax^2 + b$ (b) $a\, e^{bx^2}$
   (c) $a\, e^{-bx}$ (d) $a\, e^{bx}$
   where a and b are non-zero constants.

2. What is the area bounded by the curves $y = ln\, x$ and $y = (ln\, x)^2$?
   (a) e – 1 (b) e – 2
   (c) 3 – e (d) e

For the next three (03) items that follow:
Consider the function

$$f(x) = \int_1^x \{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\}\, dt.$$

3. The function attains local maximum at
   (a) x = 0 (b) x = 1
   (c) x = 2 (d) x = 4

4. What is the local maximum value of the function?
   (a) 0 (b) 1
   (c) 4 (d) 16

5. Consider the following statements:
   1. The function attains local minimum value at $x = \frac{7}{5}$.
   2. $x = 2$ is the point of inflexion.
   Which of the above statements is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

For the next two (02) items that follow:
Consider $f(x) = -1 + |x - 1|, -1 \le x \le 3$ and
$g(x) = 2 - |x + 1|, -2 \le x \le 2.$

6. For $x \in (0, 1), \text{fog}(x)$ is equal to
   (a) x – 1 (b) 1 – x
   (c) x + 1 (d) – x – 1

7. Consider the following statements:
   1. For $x \in (-1, 1), \text{fog}(x) = x$.
   2. For $x \in (-1, 2), \text{gog}(x) = x$.
   Which of the above statements is/are correct?
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2

For the next three (03) items that folow :

Let $f : \mathbf{R} \to \mathbf{R}$ be defined by

$f(x) = \frac{3x^2}{2} + x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0) = 0$.

8. The function f(x) is
   (a) continuous and differentiable at x = 0
   (b) nowhere continuous over **R**
   (c) continuous at x = 0, but not differentiable at x = 0
   (d) nowhere differentiable over **R**.

9. The function f(x) has
   (a) local maximum at x = 0
   (b) local minimum at x = 0 but it has no absolute minimum at x = 0
   (c) absolute minimum at x = 0
   (d) absolute maximum at x = 0.

10. Let $x_1 = \frac{1}{2n\pi}$ and $x_2 = \frac{2}{(4n+1)\pi}$ where n ∈ N. the derivative of the function f(x) attains
    (a) positive value at x_1 and negative value at x_2.
    (b) positive value at x_1 and positive value at x_2.
    (c) negative value at x_1 and positive value at x_2.
    (d) negative value at x_1 and negative value at x_2.

11. How many integral points are there within the graph of |x| + |y| < 3?
    (a) 13 (b) 15
    (c) 21 (d) 24

12. The distance of the point (4, 5) from the straight line joining the points (1, 2) and (–2, 3) measured parallel to the line x + y + 1 = 0 is
    (a) 4 units (b) $4\sqrt{2}$ units
    (c) 6 units (d) $6\sqrt{2}$ units

13. A double ordinate of the parabola $y^2 = 4ax$ is of length 8a. What is the angle between the lines from the vertex to its ends?
    (a) 30^o (b) 45^o
    (c) 60^o (d) 90^o

14. For how many values of k, the line 3x – 4y = k may touch the circle $x^2 + y^2 – 8y – 5 = 0$?
    (a) 1 (b) 2
    (c) 3 (d) None of the values of k

15. What is one of the angles between the straight lines $(x\cos\alpha - y\sin\alpha)^2 = (x^2 + Y^2)\sin^2\alpha$?
    (a) α (b) 2α
    (c) 4α (d) $\alpha/2$

16. Let S be any set and P(S) be its power set. We define a relation R on P(S) by ARB to mean $A \subseteq B$ for all A, B ∈ P(S). Consider the following in respect of the relation R :
    1. R is a reflexive relation.
    2. R is an anti-symmetric relation.
    3. R is a symmetric relation.
    4. R is a transitive relation.
    Which of the above are correct?
    (a) 1, 3 and 4 (b) 3 and 4 only
    (c) 1, 2 and 4 (d) 1 and 2 only

17. What is the real part of $(\sin x + i\cos x)^5$, where $i = \sqrt{-1}$?
    (a) $-\cos 5x$ (b) $-\sin 5x$
    (c) $\cos 5x$ (d) $\sin 5x$

18. If 5^{99} is devided by 13, then the remainder is
    (a) 1 (b) 5
    (c) 8 (d) 11

19. The number of consecutive odd integers whose sum can be expressed as $50^2 – 13^2$ is
    (a) 33 (b) 35
    (c) 37 (d) 39

20. A group of order 4 is
(a) always cyclic (b) always non-cyclic
(c) abelian and may not be cyclic (d) always non-cyclic

21. If a and b rational and $(b^2 + 1)$ is not a perfect square, then the quadratic equation with rational coefficients whose one root is $\frac{a}{2}\left(b + \sqrt{1 + b^2}\right)$ is
(a) $x^2 - 2abx - a^2 = 0$ (b) $4x^2 - 4abx - a^2 = 0$
(c) $x^2 - abx - a^2 = 0$ (d) $x^2 - abx + a^2 = 0$

22. If $2|z - 1| = |z - 2|$ and $3(x^2 + y^2) = kx$, then what is k equal to?
(a) 2/3 (b) 4/3
(c) 4 (d) 1

23. Let $a_1, a_2, a_3, \ldots$ be a sequence of real numbers such that $|a_i| = |a_{i-1} + 1|$ for $i \geq 2$ and $a_1 = 0$. If A denotes the arithmetic mean of $a_1, a_2, a_3, \ldots, a_n$ then which one of the following is correct?
(a) $2nA = a_{n+1}^2 - n$ (b) $2nA = a_n^2 - n$
(c) $2nA = a_{n+1}^2 - n - 1$ (d) $2nA = a_n^2 - n - 1$

24. If A is a non-singular matrix of order 3, then what is adj(adj A) equal to?
(a) $|A|^3 A$ (b) $|A|^2 A$
(c) $|A|\, A$ (d) A

25. If A, B and C are the angles of an isosceles triangle, then what is
$$\begin{vmatrix} 1 & 2 & 1 \\ 1 + \sin A & 2 + \sin A + \sin B & 1 + \sin C \\ \sin A\,(1 + \sin A) & \sin A\,(1 + \sin A) + \sin B\,(1 + \sin B) & \sin C\,(1 + \sin C) \end{vmatrix}$$ equal to?
(a) 0 (b) 1
(c) sin A . sin B . sin C (d) None of the above

26. Let A and B be two 3×3 matrices whose determenants are 2 and 4 respectively. What is $\det(\text{adj}(A^{-1}B))$ equal to ?
(a) $|A|$ (b) $|B|$
(c) $4|A|$ (d) $4|B|$

27. Let S be the set S = {2, 4, 6, 8, …, 20}. Define the operation $p \odot_n q$ as remainder when pq is divided by n. Then the inverse of the element 2 in $(S, \odot_{22})$ is
(a) 12 (b) 8

(c) 6 (d) 4

28. If $|z - 25i| \leq 15$ where $i = \sqrt{-1}$, then what is |max amp (z) – min amp (z)| equal to?

(a) $\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\pi - 2\cos^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{\pi}{2} + \cos^{-1}\left(\frac{3}{5}\right)$ (d) $\cos^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right)$

29. If the quadratic equation

$$x^2 - 4px + 4p(p-1) = 0$$

Where p is real, has its real roots greater than p, then p lies in the intervel

(a) $(4, \infty)$ (b) $(-\infty, -4)$

(c) $(-4, 0)$ (d) $-4, -1)$

30. What is the sum of the 10 terms of the series $\frac{2}{3} + \frac{5}{3^2} + \frac{8}{3^3} + \ldots$?

(a) $1 + \frac{3}{4}\left(1 - \frac{1}{3^9}\right) - \frac{29}{2\times 3^{10}}$ (b) $1 + \frac{3}{4}\left(1 - \frac{1}{3^8}\right) - \frac{25}{2\times 3^9}$

(c) $1 + \frac{3}{4}\left(1 - \frac{1}{3^9}\right) - \frac{25}{2\times 3^{10}}$ (d) $1 + \frac{3}{4}\left(1 - \frac{1}{3^9}\right) - \frac{29}{3^{10}}$

31. If $f(x) = \frac{x^3\sqrt{1+x^2}}{2-x}, 0 \leq x \leq 1$ and

$f(x) = \frac{x^3\sqrt{1+x^2}}{2-x}, -1 \leq x \leq 0$, then what is $\int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx$ equal to ?

(a) 4 (b) $\frac{3}{2}$

(c) 1 (d) 0

32. Let F(x) be a twice difference function with F''(x) = – F(x) and F'(x) = G(x). If H(x) = $\{F(x)\}^2$ + $\{G(x)\}^2$ and H(5) = 5, then what is H(0) equal to?

(a) 0 (b) 5

(c) 9 (d)

33. f(a) = 2, f'(a) = 1, g(a) = –1, g'(a) = 2, then what is $\lim_{x\to a}\left[\frac{g(x)f(a) - g(a)f(x)}{x-a}\right]$ equal to?

(a) -5 (b) $\frac{1}{5}$

(c) 5 (d) $-\frac{1}{5}$

34. The function f(x) = e^x $(1 - x^2)$ is

(a) increasing for $x > \sqrt{2}$

(b) decreasing for $x < \sqrt{2}$

(c) increasing for $|x - 1| < \sqrt{2}$
(d) increasing for $|x + 1| < \sqrt{2}$

35. Consider the function f : $[0, \pi] \to [0, 1]$ defined by $f(x) = \sin\left(\frac{x}{3}\right)$. The function f is
(a) one-one (b) onto
(c) both one-one and onto (d) neither one-one nor onto

36. What is $\int_0^{\sqrt{\pi}} xe^{x^2} \sin(x^2)\, dx$ equal to?
(a) $\frac{e^{\pi}+1}{2}$ (b) $\frac{e^{\pi}-1}{4}$
(c) $\frac{e^{\pi}+1}{4}$ (d) $e^{\pi} + 1$

37. If f(x) is a second order polymial (or quadratic expression in x) and
$\int_a^b f(x)dx = (a - b)(a^2 + b^2 + ab + 2)$,
then f(x) will be of the form
(a) $3x^2 + x + 2$ (b) $2x^2 - x$
(c) $-3x^2 - 2$ (d) $3x^2 + 2$

38. If $I_1 = \int_0^{\pi/2} \cos(\sin x)\, dx$, $I_1 = \int_0^{\pi/2} \sin(\cos x)\, dx$ and $I_3 = \int_0^{\pi/2} \cos x\, dx$, then which one of the following is correct?
(a) $I_1 > I_2 > I_3$ (b) $I_3 > I_2 > I_1$
(c) $I_3 > I_1 > I_2$ (d) $I_1 > I_3 > I_2$

39. If a > 1, b > 1 then the minimum value of $\log_a b + \log_b a$ is
(a) 0 (b) 2
(c) 1 (d) None of the above

40. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then which of the following is/are correct?
1. $\frac{dy}{dx} = \frac{2}{1+x^2}$ for $-1 < x < 1$
2. $\frac{dy}{dx} = \frac{2}{1+x^2}$ for $x < -1$
3. $\frac{dy}{dx} = \frac{2}{1+x^2}$ for $x < 1$
Select the correct answer using the code given below:
(a) 1 only (b) 1 and 2 only
(c) 2 and 3 only (d) 2 and 3 only

41. The equation of the curve passing through the point (0, 1) and having x^3y^{-3} as the slope of the tangent to the curve at any point (x, y) is

(a) $x^4 - y^4 + 1 = 0$ (b) $x^4 + y^4 - 1 = 0$
(c) $x^3 + y^3 - 1 = 0$ (d) $x^3 - y^3 + 1 = 0$

42. If I_1 is the integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2$ and I_2 is the integrating factor of the differential equation $\frac{dy}{dx} + y = x^{-2}$, then which one of the following is *not* correct?
(a) $I_1 I_2 = 1$ (b) $I_2 = x^2 I_1$
(c) $I_1 = x^2 I_2$ (d) $I_2 > I_1$ for $x > 1$

43. Consider the following differential equations:
1. $(x - y)\frac{dy}{dx} = 2x + y$
2. $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + 4x$
3. $2x^2y^2\frac{dy}{dx} = x^2 + y^2$
4. $\sin x\frac{dy}{dx} = \cos x$

How many of the above are homogeneous?
(a) One (b) Two
(c) Three (d) Four

44. If $f(x) = \frac{1-x}{1+x}$ where $x > 0$ and $x \neq 1$, then $f[f(x)] + f\left[f\left(\frac{1}{x}\right)\right]$ is
(a) less than 2 (b) greater than 2
(c) greater than or equal to 2 (d) equal 2

45. What is $\int_1^3 \frac{[x^2]\,dx}{[x^2 - 8x + 16] + [x^2]}$, where [•] denotes the greatest integer function, equal to?
(a) 4 (b) 3
(c) 2 (d) 1

46. A straight line passes through a fixed point (h, g). The locus of the foot of the perpendicular on it drawn from the origin is
(a) a straight line (b) an ellipse
(c) a parpabola (d) a circle

47. If the three distinct points $(t_i, 2at_i + at_i^3)$ for $i = 1, 2, 3$ are collinear, then the sum of the abscissa of the points is
(a) –1 (b) 0
(c) 1 (d) 3

48. Let (a, b) and (c, d) be two points in a plane. Any point on the line joining these points has coordinates
    (a) (a + kc, b + kd)
    (b) (ka + c, kb + d)
    (c) ((1 – k) a + kc, (1 – k) b + kd)
    (d) (a + (1 – k) c, b + (1 – k) d)
    where k is any real number.

49. The equation
    $|\vec{r}|^2 + \dot{\vec{r}} \cdot \left(2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}\right) - 10 = 0$
    represents a sphere of radius
    (a) 2 units (b) 3 units
    (c) 4 units (d) 5 units

50. What is $\int \frac{(ax+b)dx}{|ax+b|}$, where $a \neq 0, x \neq -\frac{b}{a}$, equal to?
    (a) $\frac{(ax+b)}{a} + c$ (b) $(ax + b) + c$
    (c) $|x| + c$ (d) None of thje above

For the next three (03) items that follow:
Consider a point A(–2, 3 0) above the line PQ. The line PQ passes through P(–3, 5, 2) and makes equal angles with the coordinate axes.

51. What are the coordinates of the foot of the perpendicular from A on the line PQ?
    (a) (–4, 4, 1) (b) (4, 4, 1)
    (c) (– 2, 2, 1) (d) (2, 2, 1)

52. What are the direction ratios of the line perpendicular to the line PQ?
    (a) < 2, 1, – 1 > (b) < – 2, 1, 1 >
    (c) < 4, 1, 1 > (d) < 1, 1, 1 >

53. What is the square of the perpendicular distance of the point A from the line PQ?
    (a) 4 (b) 5
    (c) 6 (d) 9

For the next two (02) items that follow:
A variable plane $\frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$ at unit distance from the origin cuts the coordinate axes at A, B and C respectively. The centroid of the triangle ABC satisfies the equation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k^2$.

54. The centroid of the triangle is at

(a) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ (b) (a, b, c)

(c) $3a, 3b, 3c)$ (d) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

55. The value of k is

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$

(c) 3 (d) 9

56. if ABCDEF is a regular hexagon with $\overrightarrow{\mathrm{AB}} = \vec{a}$ and $\overrightarrow{\mathrm{BC}} = \vec{b}$, then what is $\overrightarrow{\mathrm{CE}}$ equal to?

(a) $\vec{\mathrm{b}} - \vec{\mathrm{a}}$ (b) $\vec{\mathrm{b}} - 2\vec{\mathrm{a}}$

(c) $2\vec{\mathrm{b}} - \vec{\mathrm{a}}$ (d) $\vec{\mathrm{b}} + \vec{\mathrm{a}}$

57. If (0, 1) and (1, 0) are mid-points of the sides of a right-angled triangle, then consider the following statements:

1. (0, 0) can be the orthocentre of the triangle.
2. (1, 1) can be the orthocentre of the triangle.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Either 1 and 2 (d) Neither 1 nor 2

For the next three (03) items that follow :

The vector $\vec{\mathrm{b}} = (\tan\alpha)\hat{\mathrm{i}} - \hat{\mathrm{j}} + 2\sqrt{\sin\left(\frac{\alpha}{2}\right)}\,\hat{\mathrm{k}}$ and $\vec{\mathrm{c}} = (\tan\alpha)\hat{\mathrm{i}} + (\tan\alpha)\hat{\mathrm{j}} - 3\sqrt{\mathrm{cosec}\left(\frac{\alpha}{2}\right)}\,\hat{\mathrm{k}}$ are orthogonal and a vector $\vec{a} = \hat{\imath} + 3\hat{\jmath} + (\sin 2\alpha)\hat{k}$ makes an abtuse angle with z-axis.

58. What is/are the permissible value(s) of $\tan\alpha$?

(a) – 2 only (b) 3 only

(c) Both – 2 and 3 (d) Neither –2 nor 3

59. in which quadrant does α lie?

(a) First quadrant (b) Second quadrant

(c) Third quadrant (d) Fourth quadrant

60. What is α equal to?

(a) $(4n+1)\,\pi \pm \tan^{-1} 2$

(b) $(4n+2)\,\pi \pm \tan^{-1} 2$

(c) $(4n+1)\,\pi - \tan^{-1} 2; (4n+2)\,\pi - \tan^{-1} 2$

(d) None of the above

where n is an integer.

61. A square matrix of third order is said to be skew-symetric if
    (a) All elements of leading diagonal are zero
    (b) $a_{ij} = a_{ji}$
    (c) All elements of leading diagonal are 1
    (d) $a_{ij} = -a_{ji}$

62. The equations
    $kx + y + z = k - 1, x + ky + z = k + 1, x + y + kz = k - 1$ has no solution if
    (a) $k = 1$ only
    (b) $k \neq -2$
    (c) $k = -2$ or 1
    (d) $k = -2$ only

63. What is the value of
    $\left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{52722} + \left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{40305}$
    where $i = \sqrt{-1}$
    (a) $\sqrt{2}$
    (b) $\sqrt{3}$
    (c) 2
    (d) 3

64. The function
    $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$
    is differentiable at x = 0
    (a) only when $a_1 = 0$
    (b) only when $a_1 = a_3 = 0$
    (c) only when $a_1 = a_2 = a_3 = 0$
    (d) for any values of a_0, a_1, a_2 and a_3

65. If the complex numbers z_1, z_2, z_3 are in AP, they lie on
    (a) a circle
    (b) a line
    (c) a parabola
    (d) an ellipse

66. Which one of the following binary operations $*$ is associative on the set of real numbers?
    (a) $a * b = a^b$
    (b) $a * b = a + b - 1$
    (c) $a * b = \frac{a}{b}, b \neq 0$
    (d) $a * b = a - b$

67. All the fourth roots of unity are
    (a) $1, 1, -1, -1$
    (b) $i, i, -i, -i$
    (c) $1, -1, i, -i$
    (d) $-i, -i, -i, -i$

where $i = \sqrt{-1}$.

68. Consider the following in respect of the equation $(x + 2)^2 - 3|x + 2| = 0$:
    1. The sum of all possible rootsof the equation is – 8.
    2. The product of all possible roots of the equation is 0.

    Which of the above statements is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) neither 1 nor 2

69. Three straight lines l_1, l_2, l_3 are parallel and lie on the same plane. 5 points are taken on line l_1, 6 points are taken on line l_2 and 7 points are taken on line l_3. What is the maximum number of triangles formed with vertices at these points?
    (a) 620 (b) 746
    (c) 751 (d) 781

70. Consider the following statements in respect of the expansion $\frac{(1+x)^{2n}}{x^n}$:
    1. Independent term does not exist in the expansion.
    2. The coeficient of x is equal to cofficient of x^{-1} in the expansion.

    Which of the above statements is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) neither 1 nor 2

71. If $A \in (0, 2\pi) - \{\pi\}$, how many solutions of $\cot\frac{A}{2} - \tan\frac{A}{2} = 2$ are possible?
    (a) Only one (b) Two
    (c) Four (d) No solution is possible

72. Consider the following statements
    1. $\sin 75° + \cos 105° \neq \cos\theta$ for any θ, where $0 < \theta < 60°$
    2. $\sin\theta + \cos\theta < 1$ for all θ, where $90° < \theta < 120°$.

    Which of the above statements is/are correct?
    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) neither 1 nor 2

73. Let $T = \{\theta \in \mathbf{R} : 3\theta$ is not of the form $k\pi$ for any $k \in Z\} \cap [0, 2\pi]$.
    Consider the following statements:
    Statement I :
    There exist one $x \in \frac{B}{-1,1}$ for which there exists no $1 = \frac{2\cos 2t}{\sin 3t} = x$
    Statement II :
    For any $\theta \in T$, $\frac{1+2\cos\theta}{\sin 3\theta} = \operatorname{cosec}\theta$.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement II is the correct explanation of StatementI

(b) Both the statements are true but StatementII is not the correct explanation of Statement I

(c) Statement I is true, but Statement II is false

(d) Statement I is false, but Statement II is true

74. Consider the following statements:

Statement I :

There exist no triangle ABC satisfying $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c} = \frac{1}{2R}$, where R is the circum-radius of the triangle ABC.

Statement II :

If ABC is an isosceles triangle satisfying $b^2 = c^2 + a^2$, then $\frac{a}{\cos A} = \frac{c}{\cos C} = b$.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and StatementII is the correct explanation of StatementI

(b) Both the statements are true but StatementII is not the correct explanation of Statement I

(c) Statement I is true, but Statement II is false

(d) Statement I is false, but Statement II is true

75. Let ABC be a triangle with $\angle B = 60°$.

Statement I :

If $a = b\sin C + c\sin B$, then $\angle C \neq 45°$.

Statement II :

$b^2(1 - \sin 2C) = c^2\left(\frac{2-\sqrt{3}}{2}\right)$.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and StatementII is the correct explanation of StatementI

(b) Both the statements are true but StatementII is not the correct explanation of Statement I

(c) Statement I is true, but Statement II is false

(d) Statement I is false, but Statement II is true

76. The number 1, 2, 3, 4, 5, 6, 7, 8 are arranged in a random order. The probability that the digits 1, 2, 3, 4 appear as neighbours in that order is

(a) $\frac{1}{2}$ (b) $\frac{1}{128}$

(c) $\frac{1}{256}$ (d) $\frac{1}{336}$

77. The average marks of 10 students in a class was 60 with a standard deviation 4, while the average marks of other 10 students was 40 with a standard deviation 6. If all the 20 students are taken together, their standard deviation will be
(a) 5.0 (b) 7.5
(c) 9.8 (d) 11.2

78. The two lines of regression of y on x and x on y are 5y + 4x = 37 and y + 5x 20 respectively. The correlation between x and y will be
(a) $\frac{2}{5}$ (b) $\frac{-2}{5}$
(c) $\frac{1}{5}$ (d) $\frac{-1}{5}$

79. Correlation between two variable X and Y is given to be 0.6. These variables are transformed to new variables u = –2X + 3 and v = 5Y – 2. What will be the correlation between u and v?
(a) 0.6 (b) –0.6
(c) 0.2 (d) Information is insufficient

80. If A and B are any two events with P(A) = 0.6, P(B) = 0.3 and P(A ∩ B) = 0.2, what will be $P(A^c|B^c)$, where A^c is the complementary event of A?
(a) $\frac{3}{7}$ (b) $\frac{4}{7}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

81. A point is chosen at random inside a rectangle measuring 5 inches. What is the probability that the point chosen at random inside the rectangle is at least one inch from the edge?
(a) $\frac{5}{6}$ (b) $\frac{4}{5}$
(c) $\frac{3}{4}$ (d) $\frac{2}{5}$

82. A box contains three types of seeds : 50% of type A; 20% of type B and rest of type C. It is known that 20% of A, 30% of B and 30% of C germinate. A seed is drown randomly from the box. What is its probability to germinate?
(a) 0.25 (b) 0.50
(c) 0.80 (d) 1

83. A box contains a fair coin and a two-headed coin B. A coin is selected at random from the box and tossed twice. If head comes both the times, the probability that it is by the two-headed coin is
(a) 1/4 (b) 1/2
(c) 4/5 (d) 5/8

84. Some urns contain 4 white and 6 black balls, while one urn contains 5 white and 5 black balls. One urn is chosen at random from these and 2 balls are drawn from it, and both are found to be black. The probability that 5 white and 3 black balls remain in the chosen urn is 1/7. The total number of urns is
(a) 4 (b) 5
(c) 6 (d) 7

85. n observations on a variable X and $X_i = A + iB$ for $i = 1, 2, 3, \ldots, n$ where A, B are real constants. The mean of the observations is
(a) $A + B\left(\frac{n+1}{2}\right)$ (b) $A + Bn\left(\frac{n+1}{2}\right)$
(c) $nA + B\left(\frac{n+1}{2}\right)$ (d) $A + B\left(\frac{n}{2}\right)$

86. It is given that $(\sin^{-1} x)\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi^2}{36}$. Which one of the following is *not* correct?
(a) $\sin^{-1} x - \cos^{-1}\left(\frac{1}{2}\right) \neq 0$
(b) $\sin^{-1} x + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
(c) $\sin^{-1} x = \frac{1}{5}\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
(d) $(\sin^{-1} x)^2 = \frac{1}{9}\left[\cos^{-1}\left(\frac{1}{2}\right)\right]^2$

87. Consider the following statements:
1. If α, β are supplementary angles and $\cot(\alpha - \beta) = 1$, then $\tan 2\alpha = \cot 2\alpha$.
2. if α, β are complementary angles and $\tan(\alpha - \beta) = 1$, then $\sec 2\beta = \text{cosec } 2\beta$.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) neither 1 nor 2

88. Consider the following statements:
1. If α, β are acute angles and $\tan(\alpha - \beta) = 1$ and $\sqrt{3}\sec(\alpha - \beta) = 2$, then $\tan 2\alpha = \cot 15°$.
2. If α, β are the angles in the second quadrant and $\text{cosec}(\alpha - \beta) = -\sec(\alpha + \beta) = 2$, then $\sin\beta = \cos 15°$.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) neither 1 nor 2

89. What is the maximum value of
$5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$?
(a) 11 (b) 10
(c) 5 (d) 1

90. Consider the following statements:
1. If $\theta = -\frac{17\pi}{4}$, then $\sin^8\theta = \frac{1}{8}$.
2. If $\theta = \frac{231\pi}{6}$, then $\sin^6 3\theta = 1$.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) neither 1 nor 2

91. For how many distinct values of A between $0°$ and $360°$ is the expression $\frac{\sin A+\sin 2A+\sin 3A}{\cos A+\cos 2A+\cos 3A}$ undefined?
(a) 2 (b) 4
(c) 6 (d) 8

92. In a triangle ABC if $\cot\frac{A}{2}, \tan\frac{B}{2}, \cot\frac{C}{3}$ are in HP, what is the value of $\tan\frac{A}{2}\tan\frac{C}{2}$?
(a) -1 (b) $\frac{1}{2}$
(c) 1 (d) 2

93. If the function f(x) = sin x + cos (xa) is periodic then 'a' is
(a) always a natural number
(b) always an integer
(c) an irrational number
(d) a rational number

94. At how many points do y = x and y = tan x intersect?
(a) Zero (b) Only one
(c) Two (d) Infinite

95. ABCDEFG is a 7-sided polygon which is not regular. If its angles are in AP, then which one of the following is correct?
(a) Exactly three of its angles are greater than 125^o.

(b) Exactly four of its angles are greater than or equal to the angle of a regular polygon of 7-sides.

(c) Exactly three of its angles are less than or equal to $\frac{5\pi}{7}$ radian.

(d) The sum of the greatest angle and the least angle is greater than $\frac{10\pi}{7}$ radian.

96. If $x = \varphi(t), y = \psi(t)$, then what is $\frac{d^2y}{dx^2}$ equal to ?

(a) $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^2}$ (b) $\frac{\varphi'\psi'' - \psi'\varphi''}{(\varphi')^3}$

(c) $\frac{\varphi''}{\psi''}$ (d) $\frac{\varphi'\psi'' + \psi'\varphi''}{(\varphi')^2}$

where dashes denotes the derivative with respect to t.

97. Let f(x) = sin x, g(x) = x^2 and h(x) = ln x be functions of real variable x > 0. Suppose fog(x) means f[g(x)]. If F(x) = [(hog)of](x), what is F''(x) equal to ?

(a) 2 $cosec^2$ x (b) 2 sec^2 x

(c) – 2 $cosec^2$ x (d) None of the above

98. If f(x) = a ln |x| + bx^2 + x has its extreme valeus at x = – 1 and x = 2, then what is the value of 'a'?

(a) 1 (b) 2

(c) – 1 (d) – 2

99. If g(x) = x^3 and 3 f(x) = $4x^3$ – 12x where $0 \leq x \leq 2$, then g[f(x)]will attain its greatest value at

(a) x = 2 (b) x = 0

(c) x = 1 (d) x = $\frac{1}{3}$

100. If 5y = - 3[x] + 4[tan x] + 3|y| where [.] is the greatest integer function, then y as a function of x is

(a) not continuous at x = 0

(b) continuous at x = 0

(c) differentiable at x = 0

(d) continuous at x = 0 but not differentiable at x = 0