PART-I (1 Mark)

MATHEMATICS

1. Let $s_1(n)$ be the sum of the first n terms of the arithmetic progression $8, 12, 16, \ldots$, and let $s_2(n)$ be the sum of the first n terms of arithmetic progression $17, 19, 21, \ldots$. If for some value of n, $s_1(n) = s_2(n)$ then this common sum is

   A not uniquely determinable **B** 260 **C** 216 **D** 200

2. The number of x in $[0, 2\pi]$ such that $(\sin(2x))^4 = 1/8$ is

   A 2 **B** 4 **C** 8 **D** 16

3. The set of real numbers x satisfying

$$\frac{3x^2 - 8x + 5}{4x^2 - 3x + 7} > 0$$

   is

   A the set of all real numbers

   B the set of all positive real numbers

   C the set of all real numbers strictly between 1 and 5/3

   D the set of all real numbers which are either less than 1 or greater than 5/3

4. If a, b and c are distinct real numbers such that $a : b + c = b : c + a$ then

   A a, b, c are all positive **B** a, b, c are all negative

   C $a + b + c = 0$ **D** $ab + bc + ca + 1 = 0$

5. How many positive real numbers x are there such that $x^{x\sqrt{x}} = (x\sqrt{x})^x$?

   A 1 **B** 2 **C** 4 **D** infinite

6. Let $[x]$ denote the greatest integer part of a real number x. If

$$M = \sum_{n=1}^{40} \left[\frac{n^2}{2} \right],$$

   then M equals

   A 5700 **B** 5720 **C** 5740 **D** 5760

7. Suppose for fixed real numbers a and b, $f_c(x) = x^3 + ax^2 + bx + c$ has 3 distinct roots for $c = 0$. Then

   A $f_c(x)$ has 3 distinct roots for all real c

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B $f_c(x)$ has 3 distinct roots for all real $c > 0$ or for all real $c < 0$, but not for both

C $f_c(x)$ has 3 distinct roots for all real c in (p, q) for some $p < 0$ and $q > 0$

D $f_c(x)$ need not have 3 distinct roots for any real c other than zero

8. Given a semicircle of radius 1, let a be the side of an equilateral triangle which is inscribed in the semicircle with its vertices on the boundary of the semicircle (boundary includes the bounding diameter also). Then the set of possible values of a is

A $\{1\}$

B $\{1, 2/\sqrt{3}\}$

C the set of all positive real numbers not exceeding $2/\sqrt{3}$

D the set of all real numbers which are greater than or equal to 1, but less than or equal to $2/\sqrt{3}$

9. A ray of light originating at the vertex A of a square $ABCD$ passes through the vertex B after getting reflected by BC, CD and DA in that order. If θ is the angle of the initial position of the ray with AB then $\sin\theta$ equals

A $2/\sqrt{13}$ **B** $3/\sqrt{13}$ **C** $3/5$ **D** $4/5$

10. Let ABC be triangle with $AB = AC = 6$. If the circumradius of the triangle is 5, then BC equals

A $25/3$ **B** 9 **C** $48/5$ **D** 10

11. Let D and E be points on sides AB and AC of a triangle ABC such that

(a) DE is parallel to BC;

(b) DE divides the area of the triangle ABC into two equal parts.

Then the ratio of the distance from A to DE to the distance between DE and BC is

A $1 : 1$ **B** $2 : 1$ **C** $\sqrt{2} : 1$ **D** $\sqrt{2} + 1 : 1$

12. If α and β are acute angles such that $\cos^2\alpha + \cos^2\beta = 3/2$ and $\sin\alpha \cdot \sin\beta = 1/4$, then $\alpha + \beta$ equals

A $30°$ **B** $45°$ **C** $60°$ **D** $90°$

13. There is a point inside an equilateral triangle which is at distances 1, 2 and 3 from the three sides. The area of the triangle is

A not uniquely determinable **B** 6

C $6\sqrt{3}$ **D** $12\sqrt{3}$

14. The wealth of a person A equals the sum of that of B and C. If he distributes half of his wealth between B and C in the ratio $2:1$ then the wealth of B equals the sum of that of A and C. Then the fraction of the wealth that A should distribute between B and C in the ratio $1:2$ so that the wealth of C equals the sum of that of A and B is

    A 1/2 **B** 2/3 **C** 3/4 **D** 1

15. You have a measuring cup with capacity 25 ml and another with capacity 110 ml. The cups have no markings showing intermediate volumes. Using a large container and as much tap water as you wish, what is the smallest amount of water you can measure accurately?

    A 1 ml **B** 5 ml **C** 10 ml **D** 25 ml

16. A person travels in a car from his village to a town at a speed of 50 kms/hr for four hours. There he spends an hour moving about 20 kms. He then returns to his village at a speed 40 kms/hr. His average speed over the whole journey (in kms/hr) is

    A 44 **B** 42 **C** $44\frac{4}{9}$ **D** $46\frac{2}{3}$

17. There are 20 unit cubes all of whose faces are white, and 44 unit cubes all of whose faces are red. They are put together to form a bigger cube $(4 \times 4 \times 4)$. What is the minimum number of white faces visible on this larger cube?

    A 20 **B** 14 **C** 12 **D** 8

18. The number of 2-digit numbers n such that 3 divides $n-2$ and 5 divides $n-3$ is

    A 5 **B** 6 **C** 7 **D** 10

19. Let m be the number of ways in which two couples can be seated on 4 chairs in a row so that no wife is next to her husband and n be the number of ways in which they can be seated in 4 chairs in a circle. In the latter case, rotations are considered different configurations. Then

    A $m = n$ **B** $m = 2n$ **C** $m = 4n$ **D** $m = 8n$

20. A certain school has 300 students. Every student reads 5 newspapers and every newspaper is read by 60 students. Then the number of newspapers

    A is at least 30 **B** is at most 20

    C is exactly 25 **D** cannot be determined by the data